Frozen Singularities and Moduli Spaces in High Dimensions

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Based on upcoming works
[HP] and [Montero, HP]

IPhT SACLAY

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In [de Boer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi 2001] the moduli space of string vacua with 16 supercharges was studied very thoroughly, specially in seven dimensions:

Heterotic description	Orientifold description	M theory on K3 with frozen singularities of type	F theory compactified on
“standard component”	$(-^8)$	smooth $K3$	$K3 \times S^1$
$\mathbb{Z}_2$ triple CHL string no vector structure	$(-^6, +^2)$	$D_4 \oplus D_4$	$(K3 \times S^1)/\mathbb{Z}_2$
$\mathbb{Z}_3$ triple		$E_6 \oplus E_6$	$(K3 \times S^1)/\mathbb{Z}_3$
$\mathbb{Z}_4$ triple		$E_7 \oplus E_7$	$(K3 \times S^1)/\mathbb{Z}_4$
$\mathbb{Z}_5$ triple		$E_8 \oplus E_8$	$(K3 \times S^1)/\mathbb{Z}_5$
$\mathbb{Z}_6$ triple		$E_8 \oplus E_8$	$(K3 \times S^1)/\mathbb{Z}_6$
	$(-^4, +^4)_1$	$(D_4)^4$	$(T^4 \times S^1)/\mathbb{Z}_2$
	$(-^4, +^4)_2$		
		$(E_6)^3$	$(T^4 \times S^1)/\mathbb{Z}_3$
		$D_4 \oplus E_7 \oplus E_7$	$(T^4 \times S^1)/\mathbb{Z}_4$
		$D_4 \oplus E_6 \oplus E_8$	$(T^4 \times S^1)/\mathbb{Z}_6$

Frozen singularities are such that they cannot be resolved, thus they reduce the number of moduli and gauge group rank. In this case, due to nontrivial 3-form background. Not well understood.

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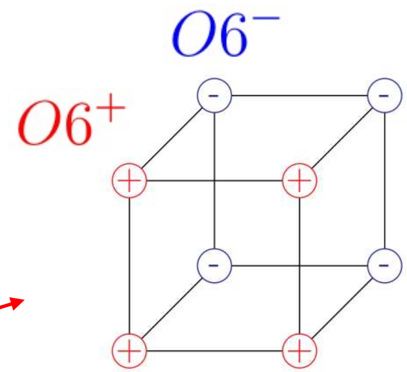
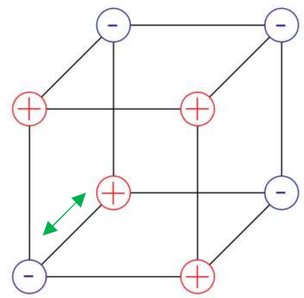


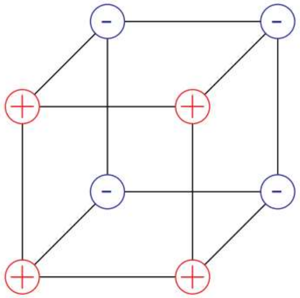


The highlighted orientifolds are perturbatively inequivalent. Nonperturbative inequivalent if there exist inequivalent embeddings (uplifts to K3)

$$4\,D_4 \hookrightarrow \Gamma_{3,19}$$

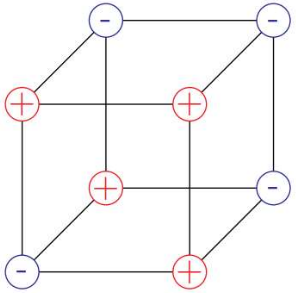
To see this, in both cases Exchange ("unfreeze") $O6^+ \longrightarrow O6^- + 4D6$

This is usual Type II orientifold, dual to Spin(32)/Z_2 Heterotic on T^3



$$A_1 = (0,0,0,0,\ 0,0,0,0,\ \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\ \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2})\,,$$
$$A_2 = (0,0,0,0,\ \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\ 0,0,0,0,\ \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2})\,,$$
$$A_3 = (0,0,0,0,\ 0,0,0,0,\ 0,0,0,0,\ 0,0,0,0)\,.$$

$W_{Spin(8)^4/\mathbb{Z}_2^2}$



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$$A_3 = (\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\ 0,0,0,0,\ 0,0,0,0,\ 0,0,0,0)\,.$$

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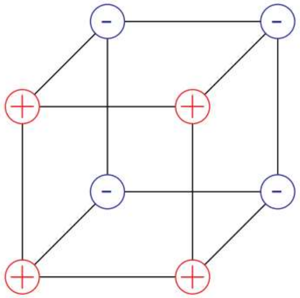

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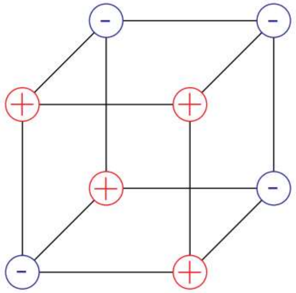
$W_{Spin(8)^4/\mathbb{Z}_2^2}$



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$$A_3 = (\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\ 0,0,0,0,\ 0,0,0,0,\ 0,0,0,0).$$

$W_{Spin(8)^4/\mathbb{Z}_2}$

$$\Lambda_{charge} = W^{\perp}_{Spin(8)^4/\mathbb{Z}_2^2} = \Gamma_{1,1} \oplus \Gamma_{2,2}(2), \qquad G_{enh} = SU(2)^3/\mathbb{Z}_2^2$$
$$\Lambda'_{charge} = W^{\perp}_{Spin(8)^4/\mathbb{Z}_2} = \Gamma_{3,3}(2), \qquad G_{enh} = SU(2)^3$$

Similarly, there are two inequivalent embeddings for $3E_6$ and $2E_7+D_4$, [Fraiman, HP 2021]

Hence **two different moduli space components** for each:

$3E_6$:

$$\Lambda_{charge} = W^{\perp}_{E_6^3/\mathbb{Z}_3} = A_2(-1) \oplus \Gamma_{1,1}, \qquad G_{enh} = SU(2)$$

$$\Lambda'_{charge} = W^{\perp}_{E_6^3} = A_2(-1) \oplus \Gamma_{1,1}(3), \qquad G_{enh} = SU(2)$$

$2E_7+D_4$:

$$\Lambda_{charge} = W^{\perp}_{E_7^2 \times Spin(8)/\mathbb{Z}_2} = 2\,A_1(-1) \oplus \Gamma_{1,1}, \quad G_{enh} = SU(2)$$

$$\Lambda'_{charge} = W^{\perp}_{E_7^2 \times Spin(8)} = 2\,A_1(-1) \oplus \Gamma_{1,1}(2), \quad G_{enh} = SU(2)$$

There is a similar story in 8D, using F-Theory on elliptic K3.

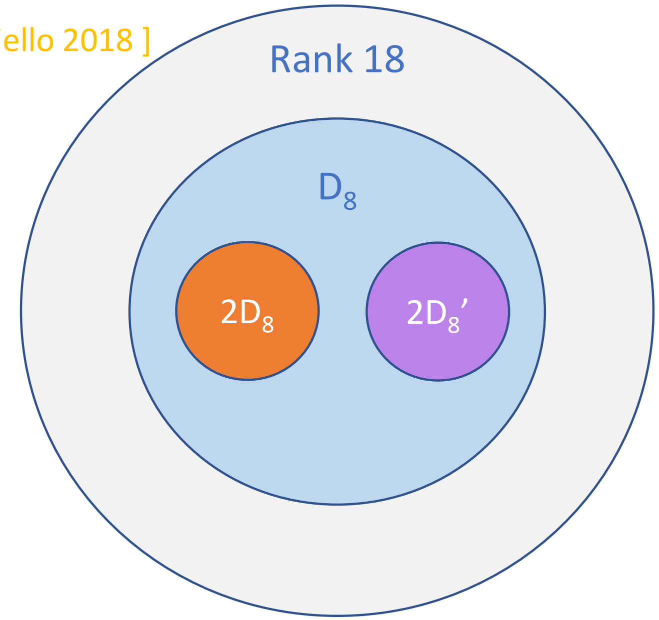
Singular fibers of type D_8 can be frozen [Witten 1998], [Bhardwaj, Morrison, Tachikawa, Tomasiello 2018]

For $2D_8$ there are two possibilities:

$$\Lambda_{charge} = W^{\perp}_{Spin(16)^2/\mathbb{Z}_2} = \Gamma_{1,1} \oplus \Gamma_{1,1}(2), \quad G_{enh} = SU(2)^2/\mathbb{Z}_2$$

$$\Lambda'_{charge} = W^{\perp}_{Spin(16)^2} = \Gamma_{2,2}(2), \quad G_{enh} = SU(2)^2$$

(Enhancements agree with [Cvetic, Dierigl, Lin, Zhang 2022])



These two rank 2 components have been taken to be the same in the literature. They are not.

These frozen singularity pairs correspond to $4D_4$ in 7D just discussed, $2D_4$ embeds into affine D_8 [Witten 1998]

Natural proposal: D_8 and $2D_8$ uplift to 9D frozen singularities, possibly in real elliptic K3.

[Cachazo, Vafa 2000]

$$D_8 \to E_8\,, \qquad 2\,D_8 \to 2\,E_8 \text{ or } D_{16}\,, \qquad 2\,D'_8 \to D'_{16}$$

Natural proposal: D_8 and $2D_8$ uplift to 9D frozen singularities, possibly in real elliptic K3.

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$$D_8 \to E_8\,, \qquad 2\,D_8 \to 2\,E_8 \text{ or } D_{16}\,, \qquad 2\,D_8' \to D_{16}'$$

Two rank 2 components in 8D decompactify to:

$$\tilde{\Lambda}_{charge} = W^{\perp}_{2E_8} = \Gamma_{1,1}, \qquad G_{enh} = SU(2)$$

$$\Lambda_{charge} = W^{\perp}_{Spin(32)/\mathbb{Z}_2} = \Gamma_{1,1}, \qquad G_{enh} = SU(2)$$

$$\Lambda'_{charge} = W^{\perp}_{Spin(32)} = \Gamma_{1,1}(2), \qquad G_{enh} = SU(2)$$

Described e.g. by M-Theory on KB, Type I’ on S1 with O8+ and O8- (see e.g. [Aharony, Komargodski, Patir 2007]) and same Type I’ with discrete theta angle [Montero, HP, to appear]

Conclusion: updated picture (complete)

rank reduction	$d=9$	$d=8$	$d=7$
8	E_8	$Spin(16)$	$Spin(8)^2$
16	E_8^2	$\frac{Spin(16)^2}{\mathbb{Z}_2}$	$\frac{Spin(8)^4}{\mathbb{Z}_2\times\mathbb{Z}_2}$
	$\frac{Spin(32)}{\mathbb{Z}_2}$		
	$Spin(32)$	$Spin(16)^2$	$\frac{Spin(8)^4}{\mathbb{Z}_2}$
12	-	-	E_6^2
14	-	-	E_7^2
16	-	-	E_8^2
	-	-	$E_8^{2\prime}$
18	-	-	$\frac{E_6^3}{\mathbb{Z}_3}$
	-	-	E_6^3
	-	-	$\frac{Spin(8)\times E_7^2}{\mathbb{Z}_2}$
	-	-	$Spin(8)\times E_7^2$
	-	-	$Spin(8)\times E_6\times E_8$

All compactifications to 6D are contained in partial classification using alternative methods. [Fraiman, HP] (to appear)

Thank You!